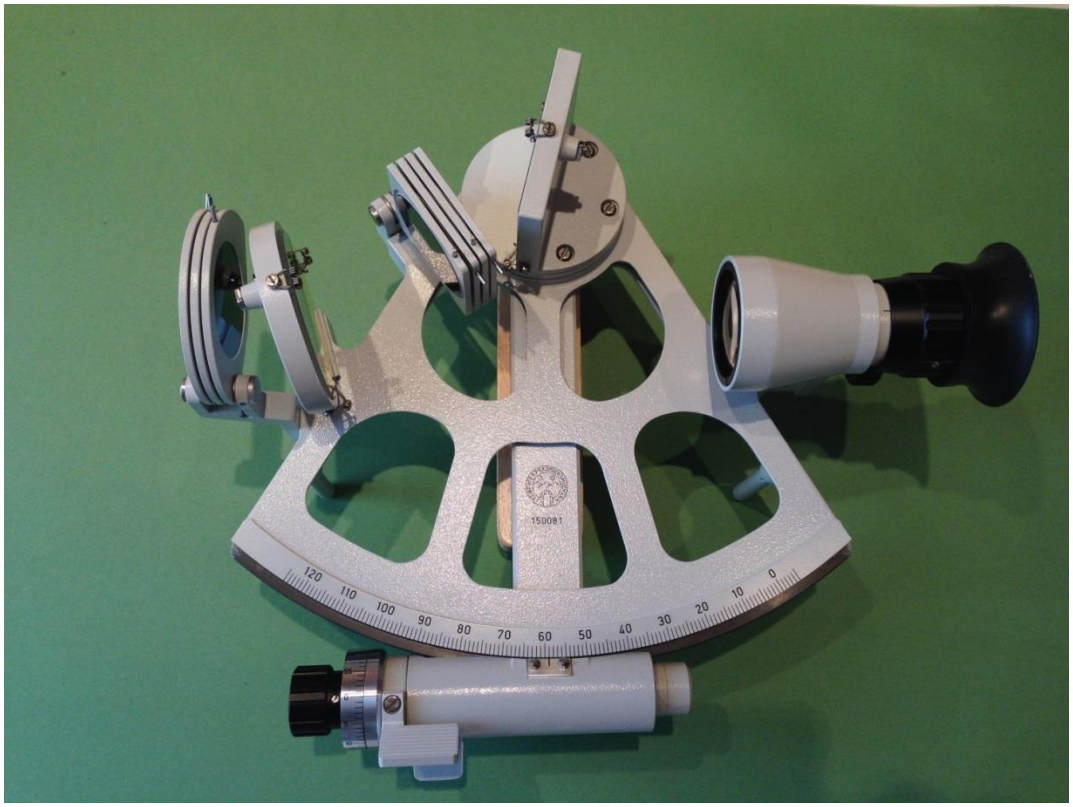
Celestial Navigation

with

Sextant and Calculator HP-42S / Free42



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Abbreviations and symbols

$LOP1$ = Line of position 1

$LOP2$ = Line of position 2

$COP1$ = Circle of position 1

$COP2$ = Circle of position 2

$H1$ = Observed altitude (Ho) in $COP1$

$H2$ = Observed altitude (Ho) in $COP2$

Dec = Declination

GHA = Greenwich Hour Angle

$D1$ = Dec of the sun in $COP1$

$G1$ = GHA of the sun in $COP1$

$D2$ = Dec of the sun in $COP2$

$G2$ = GHA of the sun in $COP2$

φ = Latitude of the FIX in the calculations of the COP

L = Longitude of the FIX in the calculations of the COP

$\Delta\varphi$ = Displacement in latitude of the observer

ΔL = Displacement in longitude of the observer

lat_M = Mean latitude

$latx$ = Dead-reckoning or assumed observer's latitude

$lonx$ = Dead-reckoning or assumed observer's longitude

FIX = Calculated position

laf = FIX latitude

lof = FIX longitude

$IN1, IN2$ = Intercept of LOP1, LOP2

$AZ1, AZ2$ = Azimuth of LOP1, LOP2

Ha = Apparent altitude

Dog = Distance over Ground of the observer

Cog = Course over Ground of the observer

Ie = Index error of the sextant

$Corr$ = Correction of the sextant height according to its certificate

HoE = Height of eye of the observer

$Da.Mo$ = Day and Month of measurement

$Year$ = Year of measurement

$U.T$ = Universal Time of measurement

$ALTs$ = Measured sextant height

$Limb$ = Top edge, bottom edge or center line of the sun

$Npos$ = Northern intersection of the circles of equal altitude

$Spos$ = Southern intersection of the circles of equal altitude

$Tran$ = Noon latitude of the observer (Transit)

Foreword

The astronomical positioning by the simultaneous measurement of a number of stars leads to a fairly accurate position, but the exercise necessary to identify those stars and to determine their height above the horizon at twilight will often be absent. That is why we limit ourselves to determine our position with the help of the sun.
Using nautical almanacs such as the Macmillan and the HO249 tables, we can traditionally determine our position on the basis of sun's altitudes.
With the advent of the programmable calculators, the entire search, calculation and plot work that follows could be taken over from us. *GHA* and *Dec* of the sun can be calculated with a built-in algorithm, as well as the azimuth and intercept of the lines of position. The somewhat difficult chart work is also digitally reproduced with the final result our position in latitude and longitude *(FIX).*

HP-42S

The HP-42S RPN Scientific is a programmable RPN calculator launched by HP in 1988. Its production was discontinued in 1995 but it is still regarded today as one of the best ever made in terms of quality and ease of programming. Its popularity was so great that he is offered on the site of Thomas Okken under the name "**Free42**: An HP-42S Calculator Simulator" as freeware for many operating systems of *computers and smartphones*. The Free42 computer application is delivered as a zip file in decimal and binary version but it is recommended to install Free42Decimal.exe. Android and iOS applications are in decimal version.
The Free42 app has the advantage that the programs can be saved and loaded as files, backups can be made and loaded into other computers. Instructions can be found here. Moreover, the calculation speed is many times higher than with the hardware version of the HP-42S.
Free42 programs are saved with the file extension ".raw".
In the astronavigation programs described below, only a limited number of keys is used, so that no thorough knowledge of the calculator is required. Those who want to deepen further can go here.

Programs

This text describes two programs for Free42 and HP-42S to navigate by the sun by means of a sextant or theodolite.
They can be downloaded from the site https://thomasokken.com/free42/42progs/.

ASTRO_TSO.raw: calculates the position *(FIX)* using the azimuth-intercept method of Marcq St. Hilaire. This method, developed in 1875, gradually became the standard procedure for astronomical positioning and is based on the choice of an assumed position (A.P), as close as possible to the true position or the estimated position, and the calculation of the intersection of two so-called *lines of position* (LOP further in the text). This program makes the use of plotting sheets superfluous.

ASTRA_TSO.raw: calculates the position *(FIX)* using an alternative method that directly calculates the intersections of two *circles of position* (COP further in the text). For a *stationary* observer, contrary to the previous method, no assumed position (A.P) is required so that one can determine his position without any idea of where one is. In the case of a *moving observer*, as in the previous program, the latitude of the observer, $latx$, must be known for the calculation of the displacement in longitude ΔL.
Both programs have a built-in almanac of the sun with a maximum deviation of ± 0.5' and an average deviation of ± 0.3' for *GHA* and *Dec*. For verification, the almanac data on the sites backbearing.com and siranah.de can be consulted.

Corrections to the measured sextant height $ALTs$

A number of corrections are applied to the measured sextant height in both programs;

- *Instrument correction:* if the sextant is provided with a certificate, this correction can be taken into account for with $Corr$ (in seconds). If not, the input is = 0
- *Index correction:* is a correction for the index error Ie of the sextant which in principle has to be measured before each new observation. Ie is negative if "off the arc" and positive "on the arc".
- *Dip:* this is a correction that is function of the height of eye HoE. When using an artificial horizon $HoE = 0$.

$$Dip[']\approx 1.76\cdot\sqrt{HoE[m]}$$

- *Refraction:* this is a correction for the atmospheric refraction of the light according to Bennett's formula.

$$Refraction[']=\frac{1}{\tan\left(Ha[°]+\frac{7.31}{Ha[°]+4.4}\right)}$$

- *Parallax:* as the horizontal parallax of the sun is very small (maximum ± 0.15'), no correction for parallax is applied.
- *Semi diameter (S.D):* this correction is made if the bottom or top edge *(Limb)* of the sun is measured. The *S.D* is calculated from the mean anomaly *(AM)* of the sun with the formula

$$S.D[°]=\frac{0.267}{1-0.017\,.\cos AM}$$

Sextant heights

$Ha = ALTs \pm Ie - Dip$ (Apparent altitude)

$Ho = Ha - Refraction \pm S.D$ (Observed altitude)

$HoE = 0$ implies the use of an artificial horizon

When using an artificial horizon $Dip = 0$ and $Ha = (ALTs \pm Ie)/2$

Used formulas

- If the Cog and Dog are known then the observer's displacement in latitude and longitude are respectively:

$$\Delta\varphi[°] = Dog[nm]\ .\ \cos Cog\ /60$$

$$\Delta L[°] = \frac{Dog[nm]\ .\ \sin Cog}{\cos lat_M\ .\ 60}$$

in which the mean latitude

$$lat_M = latx + \Delta\varphi/2$$

and $latx$ is the (assumed) latitude before displacement.

- In *ASTRO_TSO.raw* (azimuth-intercept method) the *FIX*-coordinates are

$$laf = \frac{IN2\ .\ \sin AZ1 - IN1\ .\ \sin AZ2}{\sin(AZ1 - AZ2)} + latx$$

$$lof = \frac{IN1\ .\ \cos AZ2 - IN2\ .\ \cos AZ1}{\sin(AZ1 - AZ2)\ .\ \cos latx} + lonx$$

where $latx$ and $lonx$ are the coordinates of the A.P after displacement. These values are calculated from the A.P before displacement, Cog and Dog.

Format and units of the input data

latx, lonx		+ (N and E) − (S and W)
Ie	x.xx [minutes]	− OFF the arc + ON the arc
HoE	xx.xx [meter]	Artificial horizon HoE=0
Da.Mo	xx.xx [day.month]	Ex: 5.07 23.12
U.T	xx.xx xx [h.min sec]	Ex: 17.0533 6.0806
latx, lonx, ALTs	xxx.xx x [ddd°mm.m']	Ex: −15.416 156.093
Limb	Lower=−1 Center=0 Upper=1	
Dog	xxx.x [nautical miles]	
Cog	xxx.x [degrees]	
Corr	xx [seconds]	Ex: −17 12

Program components

ASTRO_TSO.raw: consists of 3 subroutines; *LOP1, LOP2*, and *TRAN*

- *LOP1 (Line of position1):* here the assumed position (A.P), which is usually the dead-reckoning position, and the data of the first observation are entered. The subroutine calculates the intercept *IN1* and azimuth *AZ1* of the first line of position *LOP1* and stores them in their variables with the same name.

- *LOP2 (Line of position2):* on the basis of the *Dog* and *Cog* a new A.P is defined. Together with the data of the second observation, the *IN2* and *AZ2* of *LOP2* are calculated relative to this new A.P. The coordinates of the intersection of the "offset" *LOP1* and *LOP2*, being the *FIX*, are obtained with the above formulas. From a geometric point of view this actually amounts to the translation of *LOP1* in the direction (= *Cog*) of the new A.P over a distance = *Dog*. The coordinates of the *FIX* are stored as "laf"and "lof".
  Since the intercept method ignores the curvatures of the actual position lines, the obtained *FIX* is not our exact position but rather an improved position (compared to AP). The residual error remains tolerable as long as the radii of the circles of equal altitude are not too small and AP is not too far from our actual position. The geometric error inherent to the intercept method can be decreased by iteration, i.e., substituting the obtained *FIX* for AP and repeating the calculations with the same altitudes and GP's (see appendix). This will result in a more accurate position. If necessary, we can reiterate the procedure until the obtained position remains virtually constant (rarely needed).

- *TRAN (Transit):* this part calculates the noon latitude of our position. The input of the estimated longitude $lonx$ only results in a rough estimate of the time of transit because it is calculated on the basis of a fixed transit time of the Greenwich meridian (12h U.T). It is more accurate to start from the actual "Greenwich meridian transit time" and to add 4 minutes per degree of western length or subtract 4 minutes per degree of eastern length.
  The input data refer to the moment of culmination of the sun.
  "South" and "North" indicate the position of the sun relative to the observer.
  The noon latitude is stored as "laf".

Notes

- *LOP1* and *LOP2* must be entered chronologically

- no Dip correction may be applied when using a bubble sextant and/or a theodolite. To make a distinction with the use of an artificial horizon *(HoE = 0)* and to prevent *Ha* from being divided by 2, set ***HoE = 0.0001*** and the Dip correction is negligible.

ASTRA_TSO.raw : consists of 3 subroutines; *COP1, COP2,* and *TRAN*

- *COP1 (Circle of position1):* $latx$ is the estimated or known latitude of the first observation and will be used in *COP2* to calculate the displacement in longitude ΔL. For a stationary observation (*Dog = 0*) this value is irrelevant.
  After inputting the other measured values, this routine calculates the parameters that define the *Circle of position1*, i.e. *GHA (= G1), Dec (= D1)* and *Ho (= H1).*
- *COP2 (Circle of position2):* with Dog and Cog we calculate the displacement of the first *circle of position COP1*, being $\Delta\varphi$ in latitude and ΔL in longitude. With the other measured values, this routine calculates the parameters that define the *Circle of position2*, i.e. *GHA (= G2), Dec (= D2), Ho (= H2)*, and ultimately our position *(FIX).*

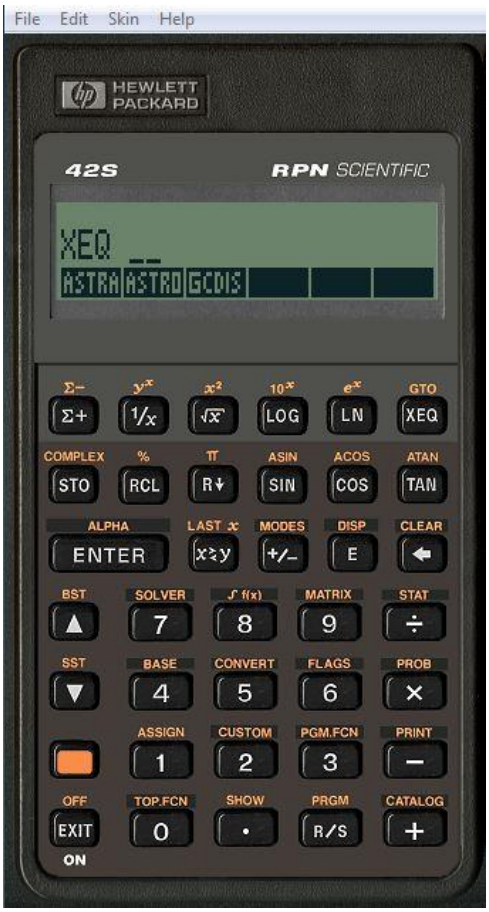
  These 2 circles normally have 2 intersections on the globe. These are our 2 possible positions. When calculating the coordinates of these points of intersection, the choice is made between the northern intersection point ($Npos$) and the southern intersection point ($Spos$). The coordinates of the *FIX* are stored as "laf" and "lof".
  The algorithm for calculating the intersections is discussed in detail in the appendix.

- *TRAN (Transit):* see *ASTRO_TSO.raw*

Notes: see *ASTRO_TSO.raw*

ASTRO* versus *ASTRA

- If one is completely in the dark about his position one can not directly enter a suitable A.P into ASTRO. In case of a *stationary measurement (Dog=0)*, ASTRA does not need this data. With 2 observations, preferably with a difference in azimuth between 30 ° and 150 °, an accurate position can be determined directly and apart from the normal measurement errors



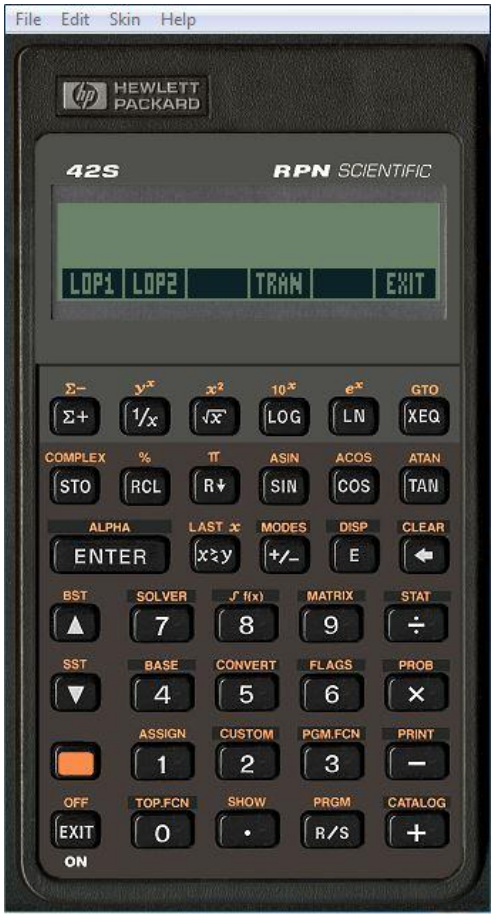
- The iteration process of the ASTRA-program runs very fast in Free42 but can take a lot of time (several seconds) with the HP-42S due to its low clock speed.

- ASTRA always iterates to the most accurate position. With ASTRO, at a great distance between the A.P and the true position and / or at large sextant heights, at least 1 iteration may be required with the calculated FIX as the new A.P.

Free42 and HP-42S commands

- Press the XEQ button. The list of programs that have been loaded appears in the display.

- Press the key below the name of the selected program and after viewing the label, press R/S. The main menu appears in the display, consisting of the 3 subroutines and the EXIT routine to close the program.



- Each routine is started with the key under its name. The requested data is entered via the number pad. Pay particular attention to the units and the format. For negative numbers, first enter the number followed by [+/-]. An incorrect input can be deleted with the [←] key and with [RTN] you return to the main menu. Each input is followed by [R/S].
In a selection menu, press the key below the desired choice. If the program stops with relevant information in the display, press [R/S] to continue.

- Entered data and calculated values are stored as variables and can be called up with [RCL] and scroll keys [▲] and [▼]

- Calculated values *GHA, Dec, laf* and *lof* are stored in the format [ddd°mm.m']. *AZ1*, *IN1* and *S.D* in [°].

- The results of the observations can be printed out. The print function is activated by [■] [PRINT] [▲] [PON] and disabled by [■] [PRINT] [▲] [POFF]. The print is available for almost all operating systems via the File menu. With the iPhone the screen must be touched just below the status bar.

Appendix: calculation of the intersections of 2 circles of position

We apply the formulas of the nautical triangle to 2 observations;

$$\sin H1 = \sin D1 \,.\sin(\varphi - \Delta\varphi) + \cos D1 \,.\cos(\varphi - \Delta\varphi) \,.\cos(G1 + L - \Delta L) \quad (1)$$

$$\sin H2 = \sin D2 \,.\sin\varphi + \cos D2 \,.\cos\varphi \,.\cos(G2 + L) \quad (2)$$

(1) and (2) are the equations of 2 circles of position in which φ and L are the geographical coordinates of the position of the observer at the time of the second measurement, i.e. the *FIX*. $H1$ and $H2$ are the true sextant heights (after corrections) and $\Delta\varphi$ and ΔL are respectively the displacement in latitude and longitude after the first measurement.

The system of 2 equations with 2 unknowns (φ and L) can not be solved analytically but by iteration. There are 2 conditions attached to this process:
- the iteration must be started in the equation of the circle with the smallest sextant height H, i.e. the circle with the largest radius.
- since for 1 value of φ, 2 values of L are possible and vice versa, the progam makes the following choice:

If *H2 ≤ H1*

then the starting value of φ (= D2) is placed in equation (2). This gives 2 possible L-values: $L = Acos(\ldots) - G2$ and $L = -Acos(\ldots) - G2$. One chooses the most eastern value and since Acos is always positive

$$L = Acos\left(\frac{\sin H2 - \sin D2\,.\,\sin\varphi}{\cos D2\,.\cos\varphi}\right) - G2$$

This L-value is placed in (1) and the new φ-value for (2) is calculated

$$\varphi = 2.Atan\left(\frac{\sin D1 \pm \sqrt{\sin^2 D1 - \sin^2 H1 + \cos^2 D1\ .\cos^2(G1 + L - \Delta L)}}{\sin H1 + \cos D1\ .\cos(G1 + L - \Delta L)}\right) + \Delta\varphi$$

with $+\sqrt{...}$ for the northern intersection and $-\sqrt{...}$ for the southern intersection.

If *H2 > H1*

then the starting value of φ (= D2) is placed in equation (1). This gives 2 possible L -values: $L = Acos(...) - G1 + \Delta L$ and $L = -Acos(...) - G1 + \Delta L$. One chooses the most western value and since Acos is always positive

$$L = -Acos\left(\frac{\sin H1 - \sin D1\ .\ \sin(\varphi - \Delta\varphi)}{\cos D1\ .\cos(\varphi - \Delta\varphi)}\right) - (G1 - \Delta L)$$

This L-value is placed in (2) and the new φ-value for (1) is calculated

$$\varphi = 2.Atan\left(\frac{\sin D2 \pm \sqrt{\sin^2 D2 - \sin^2 H2 + \cos^2 D2\ .\cos^2(G2 + L)}}{\sin H2 + \cos D2\ .\cos(G2 + L)}\right)$$

with $+\sqrt{...}$ for the northern intersection and $-\sqrt{...}$ for the southern intersection.

In both cases the value of φ is calculated by substitution, by means of the so-called T-formulas, with variables $\tan((\varphi - \Delta\varphi)/2)$ and $\tan(\varphi/2)$ respectively.

The iterations are stopped when 2 consecutive values of φ differ by less than 0.001°

The choices made in the iteration process described above implicitly assume that the earth's projection of the sun at the second observation is to the west of the earthly projection of the first observation. If it is to the east, the program will adjust these choices.

Appendix: iterations in ASTRO

For a stationary measurement *(Dog = 0)*, the calculated position can be refined by inputting the *FIX* as the new A.P with the following key sequence in the main menu:

LOP1 RCL LAF R/S RCL LOF R/S and repeating the calculations with the initial input data.

In case of a “running FIX” the new A.P is calculated with the following key sequence:

LOP1 RCL LAF R/S RCL LOF R/S RTN LOP2 R/S 180 + R/S RTN

Starting at LOP1 the calculations are repeated with the initial input data without changing the new calculated A.P values $latx$ and $lonx$.

List of equations used in the text

$$\sin H1 = \sin D1 \,.\sin(\varphi - \Delta\varphi) + \cos D1 \,.\cos(\varphi - \Delta\varphi) \,.\cos(G1 + L - \Delta L)$$

$$\sin H2 = \sin D2 \,.\sin\varphi + \cos D2 \,.\cos\varphi \,.\cos(G2 + L)$$

$$L = Acos\left(\frac{\sin H2 - \sin D2 \,.\, \sin\varphi}{\cos D2 \,.\cos\varphi}\right) - G2$$

$$\varphi = 2.Atan\left(\frac{\sin D1 \pm \sqrt{\sin^2 D1 - \sin^2 H1 + \cos^2 D1 \,.\cos^2(G1 + L - \Delta L)}}{\sin H1 + \cos D1 \,.\cos(G1 + L - \Delta L)}\right) + \Delta\varphi$$

$$L = -Acos\left(\frac{\sin H1 - \sin D1 \,.\, \sin(\varphi - \Delta\varphi)}{\cos D1 \,.\cos(\varphi - \Delta\varphi)}\right) - (G1 - \Delta L)$$

$$\varphi = 2.Atan\left(\frac{\sin D2 \pm \sqrt{\sin^2 D2 - \sin^2 H2 + \cos^2 D2 \,.\cos^2(G2 + L)}}{\sin H2 + \cos D2 \,.\cos(G2 + L)}\right)$$

$$\Delta\varphi[°] = Dog[nm] \,.\frac{\cos Cog}{60}$$

$$\Delta L[°] = \frac{Dog[nm] \,.\, \sin Cog}{\cos lat_M \;.\; 60}$$

$$lat_M = latx + \Delta\varphi/2$$

$$laf = \frac{IN2 \,.\, \sin AZ1 - IN1 \,.\, \sin AZ2}{\sin(AZ1 - AZ2)} + latx$$

$$lof = \frac{IN1 \,.\, \cos AZ2 - IN2 \,.\, \cos AZ1}{\sin(AZ1 - AZ2) \,.\, \cos latx} + lonx$$

$$Lon = \pm\, Acos\left(\frac{\sin Ho - \sin Dec \,.\, \sin Lat}{\cos Dec \,.\, \cos Lat}\right) - GHA$$

$$Dip['] \approx 1.76 \cdot \sqrt{HoE[m]}$$

$$Refraction['] = \frac{1}{\tan\left(Ha[°] + \frac{7.31}{Ha[°] + 4.4}\right)}$$

$$S.D[°] = \frac{0.267}{1 - 0.017 \,.\, \cos AM}$$

Interesting websites

http://thomasokken.com/free42/
https://www.celnav.de/page2.htm
http://www.teacupnavigation.net/CN.html
http://www.backbearing.com/almanac.htm

ASTRONAVIGATION - SUN HP 42S

```
001 LBL "ASTRO"
002 DEG
003 CF 02
004 CF 03
005 CF 29
006 "......□.ASTRO.□"
007 ├"lf.....zon"
008 AVIEW
009 STOP
010 LBL 00
011 "LOP1"
012 KEY 1 XEQ A
013 "LOP2"
014 KEY 2 XEQ B
015 "TRAN"
016 KEY 4 XEQ C
017 "EXIT"
018 KEY 6 GTO J
019 CLA
020 AVIEW
021 MENU
022 STOP
023 GTO 00
024 LBL A
025 XEQ 01
026 FC? 01
027 RTN
028 STO IN1  (°)
029 RCL 04
030 STO AZ1  (°)
031 XEQ D
032 RTN
033 LBL B
034 XEQ 10
035 FIX 01
036 INPUT Dog
037 INPUT Cog
038 COS
039 RCLx Dog
040 60
041 ÷
042 STO 23        =Δφ
043 RCL 18
044 +
045 XEQ 09
046 STO lat*
047 RCL Cog
048 SIN
049 RCLx Dog
050 RCL 18
051 RCL 23
052 2
053 ÷
054 +
055 COS
056 ÷
057 60
058 ÷
059 RCL 17
060 +
061 XEQ 09
062 STO lon*
063 XEQ 02
064 FC? 01
065 RTN
066 XEQ D
067 XEQ 03
068 SF 02
069 XEQ D
070 PRA
071 PROMPT
072 RTN
073 LBL C
074 XEQ 10
075 FIX 03
076 INPUT lon*
077 XEQ 08
078 15
079 ÷
080 12
081 X<>Y
082 -
083 >HMS
084 "Transit.."
085 PRA
086 ├".~."
087 AIP
088 ├":"
089 FP
090 100
091 x
092 AIP
093 ├".U.T"
094 PROMPT
095 XEQ 02
096 FC? 01
097 RTN
098 LBL c
099 RCL 01
100 90
101 RCL- 08
102 "SOUTH"
103 KEY 1 GTO 14
104 "NORTH"
105 KEY 4 GTO 13
106 CLA
107 MENU
108 PROMPT
109 GTO c
110 LBL 13
111 ±
112 LBL 14
113 +
114 XEQ 09
115 STO laf
116 SF 02
117 SF 03
118 XEQ D
119 CF 03
120 PRA
121 PROMPT
122 RTN
123 LBL 01
124 XEQ 10
125 FIX 03
126 INPUT lat*
127 XEQ 08
128 STO 18
129 INPUT lon*
130 XEQ 08
131 STO 17
132 LBL 02
133 FIX 02
134 INPUT Ie
135 INPUT HoE
136 XEQ I
137 INPUT Da.Mo
138 IP
139 STO 00        =JO
140 LAST X
141 FP
142 100
143 x
144 STO 15        =MO
145 RCL IND ST x
146 STO+ 00
147 FIX 00
148 INPUT Year
149 4
150 ÷
151 FP
152 STO 02        =AA
153 X=0?
154 XEQ a
155 FIX 04
156 INPUT U.T
157 >HR
158 24
159 ÷
160 STO+ 00       =J2
161 FIX 03
162 INPUT ALTs
163 FIX 00
164 INPUT Corr
165 INPUT Limb
166 RCL Year
167 2000
168 -
169 365.25
170 x
171 0.5
172 +
173 RCL- 02
174 365250
175 ÷
176 STO 14
177 RCL 00
178 LAST X
179 ÷
180 +
181 STO 01        =T
182 RAD
183 6283.01961
184 x
185 0.043179665
186 -
187 STO 02        =AM
188 XEQ 20
189 0.033417
190 RCL 02
191 SIN
192 x
193 RCL+ 02
194 RCL 02
```

```
195 2
196 X
197 SIN
198 0.0003489
199 X
200 +
201 RCL 01
202 0.300052641
203 X
204 4.938242632
205 +
206 +
207 STO 03
208 SIN
209 0.397777
210 X
211 ASIN
212 >DEG
213 STO 01          =DEC
214 XEQ 09
215 STO DEC
216 RCL 03
217 TAN
218 0.917482
219 X
220 ATAN
221 STO 02
222 RCL 03
223 COS
224 X<0?
225 XEQ b
226 RCL 00
227 360.98564735
228 X
229 RCL 14
230 360007.7
231 X
232 100.465
233 +
234 360
235 MOD           =TSO
236 +
237 360
238 ÷
239 FP
240 PI
241 X
242 2
243 X
244 RCL- 02
245 >DEG          =GHA
246 X<0?
247 XEQ 12
248 ENTER
249 XEQ 09
250 STO GHA
251 R↓
252 DEG
253 RCL lon*
254 XEQ 08
255 STO 05
256 +
257 STO 02        =LHA
258 RCL lat*
259 XEQ 08
260 STO 06
261 COS
262 RCL 01
263 SIN
264 X
265 RCL 06
266 SIN
267 RCL 01
268 COS
269 X
270 RCL 02
271 COS
272 X
273 -
274 RCL 01
275 COS
276 ±
277 RCL 02
278 SIN
279 X
280 X<>Y
281 >POL McMillan 159
282 X<>Y
283 X<0?
284 XEQ 12
285 STO 04        =AZ
286 RCL 01
287 SIN
288 RCL 06
289 SIN
290 X
291 RCL 01
292 COS
293 RCL 02
294 COS
295 X
296 RCL 06
297 COS
298 X
299 +
300 ASIN
301 STO 02        =Hc
302 RCL ALTs
303 XEQ 08
304 RCL le
305 60
306 ÷
307 -
308 RCL Corr
309 3600
310 ÷
311 +
312 RCL HoE
313 X=0?          Artif. Hor
314 GTO 16
315 SQRT
316 0.0293        =Dip
317 X
318 -
319 LBL 15
320 STO 03
321 4.4           =Refrac
322 +
323 7.31
324 X<>Y
325 ÷
326 RCL+  03
327 TAN
328 1/X
329 60
330 ÷
331 RCL 03
332 X<>Y
333 -
334 RCL Limb
335 RCL S.D
336 X
337 -
338 STO 08        =Ho
339 RCL- 02
340 STO 03        =INT
341 RTN
342 LBL 16
343 R↓
344 2
345 ÷
346 GTO 15
347 LBL a
348 1
349 STO 02
350 RCL 15
351 2
352 X>=Y?
353 RTN
354 1
355 STO+ 00
356 RTN
357 LBL b
358 PI
359 STO+ 02
360 RTN
361 LBL 03        =FIX
362 "If.....*.FIX.*"
363 AVIEW
364 RCL AZ1
365 SIN
366 RCLx 03
367 RCL 04
368 SIN
369 RCLx IN1
370 -
371 RCL AZ1
372 RCL- 04
373 SIN
374 STO 07
375 ÷
376 RCL+ 06
377 XEQ 09
378 STO laf
379 RCL 04
380 COS
381 RCLx IN1
382 RCL AZ1
383 COS
384 RCLx 03
385 -
386 RCL: 07
387 RCL 06
388 COS
389 ÷
390 RCL+ 05
391 ABS
392 180
393 X<Y?
394 GTO 11
395 LAST X
396 LBL 04
397 XEQ 09
398 STO lof
399 RTN
400 LBL D
401 CLMENU
402 CLA
403 RCL Da.Mo
404 AIP
405 ├ "/"
```

```
406 FP
407 100
408 x
409 AIP
410 ├ "/"
411 FIX 00
412 ARCL Year
413 ├ ".□."
414 RCL U.T
415 AIP
416 ├ ":"
417 FP
418 100
419 x
420 AIP
421 ├ ":"
422 FP
423 100
424 x
425 AIP
426 FS? 02
427 GTO d
428 ├ "IfAZ:."
429 ARCL 04
430 ├ "°.INT:."
431 RCL 03
432 60
433 x
434 FIX 01
435 ARCL STx
436 ├ "'"
437 PRA
438 PROMPT
439 RTN

440 LBL d

441 CF 02
442 ├ "If"
443 RCL Iaf
444 ENTER
445 XEQ 07
446 R↓
447 X<0?
448 GTO 05
449 ├ "'N.."

450 LBL e

451 FS? 03
452 RTN
453 RCL Iof
454 ENTER
455 XEQ 07
456 R↓
457 X<0?
458 GTO 06
459 ├ "'E"
460 RTN

461 LBL 05

462 ├ "'S.."
463 GTO e

464 LBL 06

465 ├ "'W"
466 RTN

467 LBL 07

468 ABS
469 AIP
470 ├ "°"
471 FP
472 100
473 x
474 FIX 01
475 RND
476 ARCL ST x
477 RTN

478 LBL 08    =>d.ddd

479 IP
480 LAST X
481 FP
482 5
483 x
484 3
485 ÷
486 +
487 RTN

488 LBL 09=>dd°mm.m

489 IP
490 LAST X
491 FP
492 3
493 x
494 5
495 ÷
496 +
497 RTN

498 LBL 10

499 CLMENU
500 SF 01
501 "RTN"
502 KEY 6 GTO H
503 MENU
504 RTN

505 LBL 11

506 LAST X
507 ENTER
508 SIGN
509 360
510 x
511 -
512 GTO 04

513 LBL 12

514 360
515 +
516 RTN

517 LBL H

518 CF 01
519 RTN

520 LBL I

521 -1
522 STO 01

523 30
524 STO 02
525 58
526 STO 03
527 89
528 STO 04
529 119
530 STO 05
531 150
532 STO 06
533 180
534 STO 07
535 211
536 STO 08
537 242
538 STO 09
539 272
540 STO 10
541 303
542 STO 11
543 333
544 STO 12
545 RTN

546 LBL 20    =>S.D

547 COS
548 0.017
549 x
550 1
551 X<>Y
552 -
553 0.267
554 X<>Y
555 ÷
556 STO S.D
557 RTN

558 LBL J

559 CLMENU
560 EXITALL
561 CLST
562 DEG
563 FIX 04
564 END
```

Sight Reduction with 2 Lines of Position (with sextant or theodolite) and calculator HP 42S or App FREE42

lat*, lon* = + (N en E)	HP 42S 1356 bytes
- (S en W)	App Free42 1353 bytes
HoE = meter	Artificial Horizon HoE=0
Limb	lower= -1 centre= 0 upper= 1
Format lat*, lon*, ALTs	= xxx.xx x [ddd°mm.m']
Format U.T	= xx.xx xx [h . min sec]
Format Da.Mo	= xx.xx [Day . Month]
Dog	= xxx.x [nautical miles]
Cog	= xxx.x [°]
Index error Ie	= - OFF the arc + ON the arc
Format Ie	= x.xx [minutes]
Format Corr	= xx [sec]

Vers. 2/12/2019

ASTRONAVIGATION - SUN HP 42S

```
001 LBL "ASTRA"
002 DEG
003 CF 02
004 CF 03
005 CF 05
006 CF 29
007 "......□.ASTRA.□"
008 ├"lf.....zon"
009 AVIEW
010 STOP
011 LBL 00
012 "COP1"
013 KEY 1 XEQ A
014 "COP2"
015 KEY 2 XEQ B
016 "TRAN"
017 KEY 4 XEQ C
018 "EXIT"
019 KEY 6 GTO J
020 CLA
021 AVIEW
022 MENU
023 STOP
024 GTO 00
025 LBL A
026 XEQ 01
027 FC? 01
028 RTN
029 STO 20    =SIN H1
030 RCL 00
031 STO 21    =GHA1
032 RCL 01
033 STO 22    =DEC1
034 XEQ D
035 RTN
036 LBL B
037 XEQ 10
038 FIX 01
039 INPUT Dog
040 INPUT Cog
041 COS
042 RCLx Dog
043 60
044 ÷
045 STO 23    =Δφ
046 RCL Cog
047 SIN
048 RCLx Dog
049 RCL 19
050 RCL 23
051 2
052 ÷
053 +
054 COS       =Mean Lat
055 ÷
056 60
057 ÷
058 STO 24    =ΔL
059 XEQ 12
060 FC? 01
061 RTN
062 STO 02    =SIN H2
063 XEQ 02
064 FC? 01
065 RTN
066 RCL 11
067 XEQ 09
068 STO laf
069 RCL 10
070 ABS
071 180
072 X<Y?
073 GTO 11
074 LAST X
075 LBL b
076 XEQ 09
077 STO lof
078 SF 02
079 XEQ D
080 PRA
081 PROMPT
082 RTN
083 LBL C     =Transit
084 XEQ 10
085 FIX 03
086 INPUT lon*
087 XEQ 08
088 15
089 ÷
090 12
091 X<>Y
092 -
093 >HMS
094 "Transit.."
095 PRA
096 ├".~."
097 AIP
098 ├":"
099 FP
100 100
101 x
102 AIP
103 ├".U.T"
104 PROMPT
105 XEQ 12
106 FC? 01
107 RTN
108 LBL c
109 RCL 01
110 90
111 RCL- 08
112 "SOUTH"
113 KEY 1 GTO 16
114 "NORTH"
115 KEY 4 GTO 15
116 CLA
117 MENU
118 PROMPT
119 GTO c
120 LBL 15
121 ±
122 LBL 16
123 +
124 XEQ 09
125 STO laf
126 SF 02
127 SF 03
128 XEQ D
129 CF 03
130 PRA
131 PROMPT
132 RTN
133 LBL 01
134 XEQ 10
135 FIX 03
136 INPUT lat*  =DRP
137 XEQ 08
138 STO 19
139 LBL 12
140 FIX 02
141 INPUT le
142 INPUT HoE
143 XEQ I
144 INPUT Da.Mo
145 IP
146 STO 00    =JO
147 LAST X
148 FP
149 100
150 x
151 STO 15    =MO
152 RCL IND ST x
153 STO+ 00
154 FIX 00
155 INPUT Year
156 4
157 ÷
158 FP
159 STO 02    =AA
160 X=0?
161 XEQ 13
162 FIX 04
163 INPUT U.T
164 >HR
165 24
166 ÷
167 STO+ 00   =J2
168 FIX 03
169 INPUT ALTs
170 RCL Year
171 2000
172 -
173 365.25
174 x
175 0.5
176 +
177 RCL- 02
178 365250
179 ÷
180 STO 14
181 RCL 00
182 LAST X
183 ÷
184 +
185 STO 01    =T
186 RAD
187 6283.01961
188 x
189 0.043179665
190 -
191 STO 02    =AM
192 XEQ 20
```

```
193 0.033417
194 RCL 02
195 SIN
196 x
197 RCL+ 02
198 RCL 02
199 2
200 x
201 SIN
202 0.0003489
203 x
204 +
205 RCL 01
206 0.300052641
207 x
208 4.938242632
209 +
210 +
211 STO 03
212 SIN
213 0.397777
214 x
215 ASIN
216 >DEG
217 STO 01          =DEC
218 XEQ 09
219 STO DEC
220 RCL 03
221 TAN
222 0.917482
223 x
224 ATAN
225 STO 02
226 RCL 03
227 COS
228 X<0?
229 XEQ 14
230 RCL 00
231 360.98564735
232 x
233 RCL 14
234 360007.7
235 x
236 100.465
237 +
238 360
239 MOD            =TSO
240 +
241 360
242 ÷
243 FP
244 PI
245 x
246 2
247 x
248 RCL- 02
249 >DEG
250 X<0?
251 XEQ 24
252 STO 00         =GHA
253 XEQ 09
254 STO GHA
255 DEG
256 FIX 00
257 INPUT Corr
258 INPUT Limb
259 RCL ALTs
260 XEQ 08
261 RCL Ie
262 60             =Ie
263 ÷
264 -
265 RCL Corr
266 3600
267 ÷
268 +
269 RCL HoE
270 X=0?       Artif.Hor
271 GTO 18
272 SQRT
273 0.0293
274 x              =Dip
275 -

276 LBL 17

277 STO 03
278 4.4            =Refract
279 +
280 7.31
281 X<>Y
282 ÷
283 RCL+  03
284 TAN
285 1/X
286 60
287 ÷
288 RCL 03
289 X<>Y
290 -
291 RCL Limb
292 RCL S.D
293 x
294 -
295 STO 08          =Ho
296 SIN
297 RTN

298 LBL 13

299 1
300 STO 02
301 RCL 15
302 2
303 X>=Y?
304 RTN
305 1
306 STO+ 00
307 RTN

308 LBL 14

309 PI
310 STO+ 02
311 RTN

312 LBL 18

313 R↓
314 2
315 ÷
316 GTO 17

317 LBL 02

318 CF 04
319 "Npos"
320 KEY 1 GTO 22
321 "Spos"
322 KEY 4 GTO 21
323 CLA
324 MENU
325 PROMPT
326 GTO 02

327 LBL 21

328 SF 04

329 LBL 22

330 CF 05
331 RCL 00
332 RCL- 21
333 ABS
334 180
335 X>Y?
336 GTO 19
337 RCL 00
338 RCL 21
339 X<Y?
340 SF 05

341 LBL 23

342 "……*.FIX.*"
343 AVIEW
344 RCL 01
345 STO 05         =STARTφ
346 SIN
347 STO 06
348 LAST X
349 COS
350 STO 07       =COS D2
351 RCL 21             =G1
352 RCL- 24
353 STO 04        =G1- ΔL
354 RCL 22
355 SIN
356 STO 08        =SIN D1
357 LAST X
358 COS
359 STO 09       =COS D1
360 RCL 02
361 RCL 20
362 X<Y?
363 GTO 04

364 LBL 03

365 RCL 02       =SIN H2
366 RCL 06       =SIN D2
367 RCL 05
368 SIN
369 x
370 -
371 RCL÷ 07
372 RCL 05
373 COS
374 ÷
375 ACOS
376 FS? 05
377 ±
378 RCL- 00
379 CPX?
380 GTO 04
381 STO 10                =L
382 RCL+ 04
383 COS
384 RCLx 09
385 STO 12
386 X↑2
387 RCL 20
388 X↑2
389 -
390 RCL 08
391 X↑2
392 +
393 X<0?
394 GTO 04
395 SQRT
396 FS? 04
397 ±
398 RCL+ 08
```

```
399 RCL 20
400 RCL+ 12
401 ÷
402 ATAN
403 2
404 x
405 RCL+ 23
406 STO 11          =φ
407 RCL- 05
408 ABS
409 0.001
410 X>=Y?
411 RTN
412 RCL11
413 STO 05
414 GTO 03

415 LBL 04

416 RCL 20          =SIN H1
417 RCL 08          =SIN D1
418 RCL 05
419 RCL- 23
420 STO 12
421 SIN
422 x
423 -
424 RCL 12
425 COS
426 ÷
427 RCL÷ 09
428 ACOS
429 ±
430 FS? 05
431 ±
432 RCL- 04
433 CPX?
434 GTO 03
435 STO 10          =L
436 RCL+ 00         =G2+L
437 COS
438 RCLx 07
439 STO 12
440 X↑2
441 RCL 02
442 X↑2
443 -
444 RCL 06
445 X↑2
446 +
447 X<0?
448 GTO 03
449 SQRT
450 FS? 04
451 ±
452 RCL+ 06
453 RCL 02
454 RCL+ 12
455 ÷
456 ATAN
457 2
458 x
459 STO 11          =φ
460 RCL- 05
461 ABS
462 0.001
463 X>=Y?
464 RTN
465 RCL11
466 STO 05
467 GTO 04

468 LBL D

469 CLMENU
470 CLA
471 RCL Da.Mo
472 AIP
473 ├ "/"
474 FP
475 100
476 x
477 AIP
478 ├ "/"
479 FIX 00
480 ARCL Year
481 ├ ".□."
482 RCL U.T
483 AIP
484 ├ ":"
485 FP
486 100
487 x
488 AIP
489 ├ ":"
490 FP
491 100
492 x
493 AIP
494 FS? 02
495 GTO d
496 ├ "lfALTs:."
497 FIX 03
498 ARCL ALTs
499 PRA
500 PROMPT
501 RTN

502 LBL d

503 CF 02
504 ├ "lf"
505 RCL laf
506 ENTER
507 XEQ 07
508 R↓
509 X<0?
510 GTO 05
511 ├ "'N.."

512 LBL e

513 FS? 03
514 RTN
515 RCL lof
516 ENTER
517 XEQ 07
518 R↓
519 X<0?
520 GTO 06
521 ├ "'E"
522 RTN

523 LBL 05

524 ├ "'S.."
525 GTO e

526 LBL 06

527 ├ "'W"
528 RTN

529 LBL 07

530 ABS
531 AIP
532 ├ "°"
533 FP
534 100
535 x
536 FIX 01
537 RND
538 ARCL ST x
539 RTN

540 LBL 08          =>d.ddd
541 IP
542 LAST X
543 FP
544 5
545 x
546 3
547 ÷
548 +
549 RTN
550 LBL 09 =>dd°mm.m
551 IP
552 LAST X
553 FP
554 3
555 x
556 5
557 ÷
558 +
559 RTN

560 LBL 10

561 CLMENU
562 SF 01
563 "RTN"
564 KEY 6 GTO H
565 MENU
566 RTN

567 LBL 11

568 LAST X
569 ENTER
570 SIGN
571 360
572 x
573 -
574 GTO b

575 LBL H

576 CF 01
577 RTN

578 LBL I

579 -1
580 STO 01
581 30
582 ST0 02
583 58
584 STO 03
585 89
586 STO 04
587 119
588 STO 05
589 150
590 STO 06
591 180
592 STO 07
593 211
594 STO 08
595 242
596 STO 09
```

```
597  272
598  STO 10
599  303
600  STO 11
601  333
602  STO 12
603  RTN

604  LBL 19

605  RCL 00
606  RCL 21
607  X>Y?
608  SF 05
609  GTO 23

610  LBL 20      =>S.D
611  COS
612  0.017
613  x
614  1
615  X<>Y
616  -
617  0.267
618  X<>Y
619  ÷
620  STO S.D
621  RTN

622  LBL 24

623  360
624  +
625  RTN

626  LBL J

627  CLMENU
628  EXITALL
629  CLST
630  DEG
631  FIX 04
632  END
```

Sight Reduction with 2 Circles of Position (with sextant or theodolite) and calculator HP 42S or App FREE42

lat*, lon* = + (N en E)	HP 42S 1444 bytes
- (S en W)	App Free42 1439 bytes
HoE = meter	Artificial Horizon HoE=0
Limb	lower= -1 centre= 0 upper= 1
Format lat*, lon*, ALTs	= xxx.xx x [ddd°mm.m']
Format U.T	= xx.xx xx [h . min sec]
Format Da.Mo	= xx.xx [Day . Month]
Dog	= xxx.x [nautical miles]
Cog	= xxx.x [°]
Index error Ie	= - OFF the arc + ON the arc
Format Ie	= x.xx [minutes]
Format Corr	= xx [sec]

Vers. 12/12/2019